Performing Calculations with Significant Digits

When multiplying and dividing numbers it is important to know how many significant digits are in each measurement you have. Let's pretend we are calculating the area of a rectangular room with the dimensions 15.778 m by 9.332 m. We find area by multiplying length times width, and the result our calculator gives is 147.240296 m^2. Now let's think about this for a second. What is the real likelihood that you can determine the square area to the millionths place from two original measurements that only went to the thousandths place? The answer is you can't! The number 147.240296 m^2 would be correct if we assumed that 15.778 m and 9.332 m were followed by an infinite number of zeroes. That is not likely the case! If there were a more precise meterstick than the first one that was used to measure the room and the dimensions were found to be 15.7781 m by 9.3325 m, wouldn't the value of the area now change too? Of course it would! When we perform multiplication and division, there is a simple rule we must follow in order to maintain a reliable answer:

The number of significant digits in your answer must have the same number of digits as the measurement that has the fewest.

What this means is that our answer 147.240296 m^2 must be rounded to 147.2 m^2 because the two original measurements (15.778 m and 9.332 m) have five and four significant digits respectively. This means that our answer must have only four significant digits (the fewer of five and four).

When you divide by an exact number (as you would in an average), that number has infinite significant digits, and therefore will never be the fewest. For example, when averaging five numbers, after summing the numbers, the division by five does not reduce your answer to one s.d. – five is an exact number here – it has infinite zeroes after it!

Some more:

(8.733)(1.22)(9.44289) = 100.6070052 becomes 101 [three s.d.]

(97.88)/(5.22)/(17.44) = 1.074676631 becomes 1.07 [three s.d.]

Try these as examples. Keep in mind for zeroes:

If the number is one or larger, any zero to the right of the decimal is significant:

34.00 is four s.d. 1.33700 is six s.d.

A bar over a zero makes all digits to its left significant:

$94\bar{0}00$ is three s.d. $940\bar{0}0$ is four s.d.

A decimal after a zero makes all digits to its left signicant:

94000. is five s.d.

(800.0)(2.131) = __________ (17433)(42.5) = __________

$$\frac{(4.040 \times 10^{5})(3.0 \times 10^{-2})}{(1.00 \times 10^{8})} = \rule{4cm}{0.4pt}$$

(5.00)(7.4) = _______ ($90\bar{0}00$)(42.500) = ________

The rules are different when adding or subtracting numbers. Here, you are allowed to keep any digit in your answer so long as each measurement has a number in that same decimal place. Let's say you are summing up the following lengths of a field to find its perimeter:

80.55 m, 70.11 m, 111.87 m, and 82.33 m.

Line the numbers as follows, noting that all the decimals are in line with one another;

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  80.55
  70.11
111.87
+82.33
344.86 m – I can keep all of these digits (five s.d. total) because every
       measurement has a value in the tenths and hundredths place.
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If someone had measured two of the lengths with a different instrument and the lengths were now:

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  80.55
  70.113
111.87
+82.334
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344.867 m – which would be rounded to 344.87 m. I cannot keep the thousandths place because the first and third measurements don't have a thousandths place. I am allowed to use it for rounding the 6 up to a 7 in the hundredths place.

Try these now:

3.10 + 0.6000 = ______________________

23770. + 945.0 = ______________________

45000 - 17500 = ______________________

0.0120 + 0.14500 = ______________________

Density Problems

Recall the formula for density, and the variety of different volume formulas to answer the following. Check significant digits and units while you work!

1. A block of aluminum is found to have a mass of 54.6 g and a volume of 20.3 cm^3. Calculate the density of the aluminum block.

2. A piece of metal is known to have a density of 16.3 g/cm^3. Once placed on the balance, it is found that the metal has a mass of 215.42 g. What is the volume of the metal?

3. The gas inside of a balloon is known to occupy a volume of 2.545×10^3 mL. It is also known that the density of the balloon is 2.31×10^{-2} g/cm^3. What is the mass of the air in the balloon?

4. A chemistry student wants to find the density of a certain rock. The rock is placed in an overflow container that empties water into a graduated cylinder. The initial volume of water in the graduated cylinder is 33.00 mL. Once the rock is placed into the container, the water level rises to 39.00 mL. The mass of the rock is found to be 101.2 g. What is the density of the rock?

5. The mass of a copper block is 5740 g. The length of the block is 12 cm and the width of the block is 4.2 cm. What is the height of the block? The density of copper is 8.96 g/cm^3.

6. Ethanol, (d = 0.89 g/mL), water, (d = 1.00 g/mL), aluminum, (d = 2.70 g/cm^3), and mercury (d = 12.88 g/cm^3) are placed inside a container. Describe what will happen to each chemical (assume no reactions take place).

7. A metal bar is believed to be made of copper. The mass of the bar is 87.43 g, and the volume of the bar is 9.80 cm^3. The density of copper is 8.92 g/cm^3. Is the bar made of copper? Why?

8. A graduate cylinder is filled with water to the 40.0 mL mark. A bolt placed in the cylinder raises the water level to the 46.0 mL mark. If the bolt has a density of 9.710 g/cm^3, what is the mass of the bolt?

9. A wooden cube is found to have a mass of 87.95 g. Its density is 5.93 g/cm^3. What is the volume of the cube? What is the length of each side?

10. What is the density of a cork with a mass of 7.45 g, and a volume of 9.3 cm^3?

11. A block has dimensions of 6.50 cm by 4.50 cm by 7.12 cm. What is the mass of the block if its density is 8.57 g/cm^3?